

# Structural Analysis of the Cantilever Construction Process in Cable-Stayed Bridges

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## Abstract

*This work deals with the analysis of cable-stayed bridges at different erection stages during construction, assuming the full or the partial cantilever method and performing multiple finite element computational procedure. The forward process and the backward process analysis are investigated and compared: the former is performed by following the sequence of erection stages in bridge construction and the latter is carried out in the reverse direction of erection procedures. The required pretension in cable-stays and the corresponding structural configurations of the bridge at different erection stages have been examined and compared in details, also by comparing either the linear computation procedure or the nonlinear computation procedure. Numerical, analytical and construction process results are presented, compared and commented upon.*

## Keywords

*cable-stayed bridge · design · construction phases*

## 1 Introduction

The first “modern” cable-stayed bridges was built in concrete by Eduardo Torroja in the 1920s (Tampul aqueduct) and by Albert Caquot in 1952 (Donze‘re canal bridge); but the real development came from Germany with the Franz Dischinger studies and with the famous series of steel bridges crossing the river Rhine [1]. The international development of this bridge type began in the 1970s, but a very big step forward took place in the 1990s, when cable-stayed bridges entered in the domain of very long spans which was previously reserved for suspension bridges [1–3]. The concept of supporting a bridge deck by inclined tension stays can be traced back to the seventeenth century, but rapid progress in the analysis and construction of cable-stayed bridges has been made over the last years: nowadays, this type of bridges are on a rapid growth mainly for the development of computer technology, high strength steel, orthotropic steel decks and construction technology [4–8]. Because of the rapid and easy erection, the cable-stayed bridge is considered as suitable for medium to long span bridges with spans ranging from 200 to about 1000m [6, 7]. The main components of cable-stayed bridges, i.e., the pylon, the deck, and the cable stays, govern the distribution of member forces more significantly as the length of a bridge becomes longer. The longer the bridge, more cables are required; and it makes the cable-stayed bridges with multiple stays highly redundant and more difficult to be analyzed with good accuracy. In spite of the difficulties involved in the analysis of a bridge, a long-span cable-stayed bridge has attracted many bridge engineers to take it in their project, thanks to the slenderness of the modern bridge girders. However, because of their huge size and non-linear structural behaviour, the analysis of cable-stayed bridges is much more complicated than that of conventional bridges, such as truss and girder bridges. The sources of non-linearity in cable-stayed bridges mainly include behaviour of the cables, and large deflection effects. A relevant amount of sources could be found on this matter in the last half century [7–12]. But few papers presented concerns with the analysis of cable-stayed bridges during construction stages: as a matter of fact, some studies concerning with the erection procedure of cable-stayed bridges focus only on improving the con-

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struction technology [8, 13, 14], but not on the analysis. Others are focused on shape finding analysis [15] or on special issues, as the dynamic response under wind loads [16]. Consequently, as the construction sequence plays an important part in the assembly of any structure, and it's of relevant importance in cable-stayed bridges [17], more specific analysis and investigations for construction procedures are needed: in fact, the amount of pretension in the cables at each stage of construction influences the final geometry under static load conditions, and therefore the dynamic characteristics of the cables and of the bridge as a whole. Even a slight deviation from the proposed erection procedure at any stage will alter the final geometry considerably, thus altering for e.g. the frequencies of vibration. Concerning the construction method, because of their self-anchored cable systems, the cantilever method has been widely used for the girder erection of cable-stayed bridges: the cantilever method is considered the more convenient solution for constructing the cable-stayed bridges of large span, where new girder segments are installed and then supported by new cable stays in each erection stage, and the construction process keeps going stage-by-stage until the bridge is completed. Since no auxiliary supports are needed for constructing the bridge girder in the cantilever method, a lot of construction cost and time can be saved. There are two basic alternatives in the cantilevering, the partial cantilever and the full cantilever method: in the former the side span girders of the bridge are erected on auxiliary piers and afterwards the stiffening girder in main span is erected by one-sided free cantilevering until the span centre or the anchor pier on the far end is reached. In the latter, the bridge girder is erected from both side of the tower towards the anchor piers and the main span centre by double-sided free cantilevering [7, 8, 14]. In this framework, the aim of this study is to present a comparison among different construction cantilever methods, the partial and the full cantilever method, by adopting both a forward process analysis and a backward analysis, and at the same time comparing either the linear computation procedure (linear theory) or the nonlinear computation procedure (nonlinear theory). Results on the studied aforementioned issues can provide useful informations for researchers and bridge engineers, on the design tasks of cable-stayed bridges, considering different approaches.

## 2 Structural analysis

### 2.1 Case study

As an example, a three span symmetric cable stayed bridge is considered, with a composite steel-concrete girder and a double plane suspension system, according to the scheme shown in Fig. 1. The deck span is 100m long. The longitudinal scheme is perfectly horizontal, and adequate slopes of the pavement in the transverse direction are designed. The composite girders is made up of three double-T elements 12.5m spaced; composite sections have been designed according to EN 1994-1-1 [18]. The two pylons are composed by H tower with three transverse beams at 50-100-150m height (Fig. 2). Concerning boundaries,

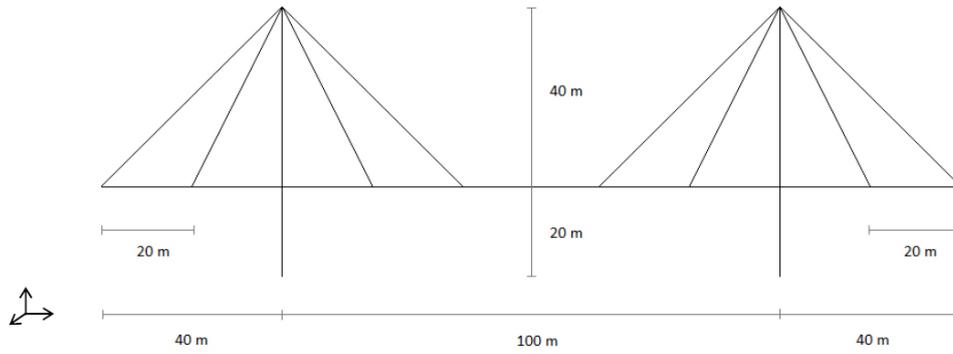
two models have been considered in relation to the restraint condition of the deck in the central span: the condition  $\alpha$  consists in two continuous decks connected by an hinge whereas in the condition  $\beta$  a continuous deck is considered (Fig. 3). It should be highlighted, that this is an illustrative example and cannot reflects all the behaviour characteristics of typical cable-stayed bridges. The analysis have been developed with MIDASoft [19], by using beam elements for the deck and the pylons and truss elements for strands, according to Wilson et al. [20]. Adopted structural materials are the following: wire strands with diameter  $\Phi = 20$  cm (ultimate stress  $f_u = 1570$  MPa, stress at 0.2% strain  $f_{y0.2\%} = 1180$  MPa), steel deck S355 J0 (ultimate stress  $f_u = 510$  MPa, yield stress  $f_y = 355$  MPa) and concrete C30/37. The bridge is fixed at the base of the pylons and simply supported at the abutments. The connection between the deck and pylons is made with an elastic link into three directions, with the following stiffness: along x axis:  $SDR_xR = 5 \cdot 10^6$  kN m; along y axis:  $SDR_yR = 10^9$  kN m; along z axis:  $SDR_zR = 10^9$  kN m. During the construction phase, temporary supports in the partial span at cable position and a rigid connection between the deck and the tower are considered. In Table 1 and 2 the material and inertial properties of the various part of the elements of the bridge are reported. Load models have been implemented according to EN 1991-2 [21].

Tab. 1. Material properties

	Elastic modulus [MPa]	Poisson ratio	Unit weight [kN/m <sup>3</sup> ]
Girder	210.000	0.30	78.5
Lower pylon	25.000	0.17	25.0
Pylon	210.000	0.30	78.5
Cables	157.000	0.30	78.5

### 2.2 Initial configuration

In cable-stayed structures, the outcome of any analysis, whether static or dynamic, depends primarily on the definition of the initial configuration, namely the structural response under permanent actions. The initial shape of a cable-stayed bridge, once defined the weight of the various elements, mainly depends on pretension force distribution in the cables. A number of techniques for the detection of initial stress distribution of pretension exist: a finite element procedure for determining the initial shape of cable-stayed bridges under the action of dead loads and pretension in inclined cables is presented for e.g. in Wang [22]. The system equation related to cable-stayed bridges including geometrical nonlinearities is usually solved using the Newton-Raphson method. Based on a reference configuration and an assumed cable pretension force, the equilibrium configuration under dead load is then found. Further, by adjusting cable forces, a 'shape iteration' is carried out and a new equilibrium configuration, i.e., a more reasonable initial shape, can be determined. The shape iteration is then repeated until the desired tolerance is achieved. Another method is for e.g. described



**Fig. 1.** Overall bridge layout (unit in metre)

**Tab. 2.** Inertial properties

	A [m <sup>2</sup> ]	Inertial moment IR <sub>xx</sub> [m <sup>4</sup> ]	Inertial moment IR <sub>yy</sub> [m <sup>4</sup> ]	Inertial moment IR <sub>zz</sub> [m <sup>4</sup> ]
Girder	0.8	15.0	1.0	15.0
Lower pylon	50.0	1000.0	500.0	500.0
Pylon	0.3	5.0	5.0	5.0
Cables	0.005	0.0	0.0	0.0

in Negroao and Simoes [23, 24]: in this case, the cable force is determined minimizing particular functions defining the overall structure from the structural and economical point of view. Other solutions are presented by Lazar et al. [25], Troitsky [26] and Gimsing [7]: the cable forces are in this case determined in order to have a particular distribution of the longitudinal bending moment along the deck. In this study, the "ULF-Unknown Load Factor" method has been chosen: this optimization technique is implemented in MIDASoft [19] and allows pre-tension forces calculations. Moreover, it is possible to calculate initial tensile forces in cables satisfying the conditions specified by the designer. Pre-determined pretension forces are chosen in order to minimize an objective function defined by one of the following:

$$\sum |w_i X_i| \quad \text{as a sum of the absolute values} \quad (1)$$

$$\sum (w_i X_i)^2 \quad \text{as a quadratic values sum} \quad (2)$$

$$\max |w_i X_i| \quad \text{as the maximum absolute values} \quad (3)$$

where

$w_i$  is the weight to be applied at the unknown load condition;

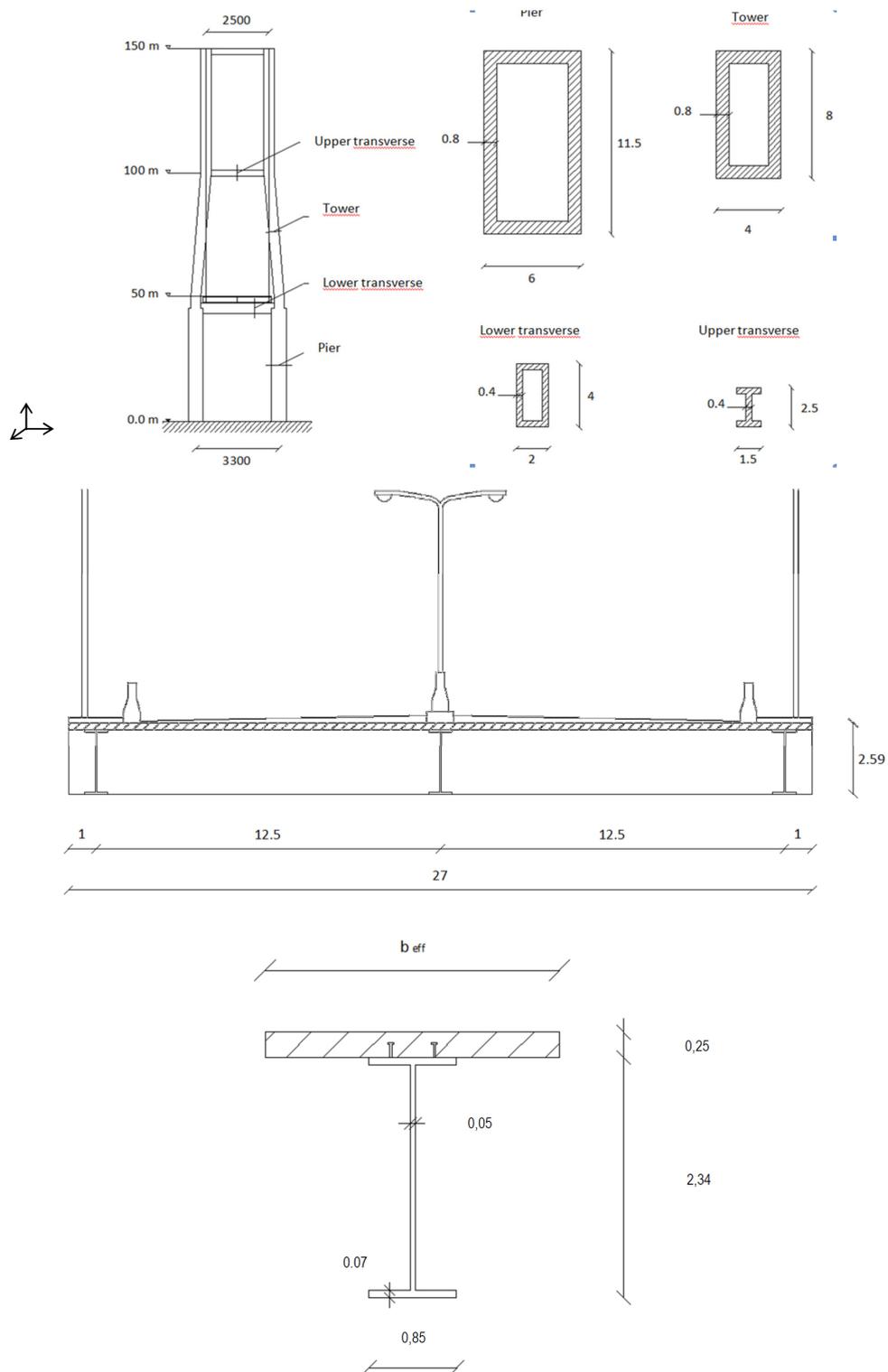
$X_i$  is the unknown load factor, in this case the pretension force in each cable.

Moreover, the boundary conditions could be imposed both on nodal displacement and on solicitation parameters, or finally on reactions. In Figure 4 are described the structural response conditions imposed on the bridge structure which are: a null displacement in longitudinal direction on the top of the pylon and fixed bending moments at specific points of the deck. Load parameters specified in the model are permanent loads, and unit forces for each cable; then, the structure is solved in linear range

for every load condition. Then the unknown pretension force is determined by adopting the superposition principle: this is associated to the external conditions imposed to the model with the quadratic sum value mode. As a matter of fact this solving method is not suitable where non elastic behavior is undergone throughout the structure, however it could be useful for preliminary calculations. The structural analyses, have been conducted in both the linear (LSA) and non-linear field, with non-linearity in cables' behavior (NLSA<sub>lg</sub>), not including geometry, and geometric non linearities (NLSA<sub>nlg</sub>). The large displacement method have been considered in the aforementioned non-linear structural analysis.

### 2.3 Construction phase

The construction advancement in cable-stayed bridges is in every moment performed by researching the equilibrium of the partial structures at various times. For three span structures, the procedure is related to the cantilever building approach from both pylons. Generally, reducing the cable distance the carriage cost decreases, but cables and anchorages cost increase hence it is necessary to rigorously determine the structure configuration in every phase. The construction is related with the cantilever method, and consequently the most relevant problem is to minimize the bending forces in the deck during erection phases, searching for the optimal cable spacing. Cantilever construction alternatives are the double cantilever method and the partial span cantilever method. The complete description of the construction phases is reported in Fig. 5 and 6, respectively for the double cantilever and for the partial cantilever method. As founded the bridge shape under the permanent loads, the problem is related to the construction method helping in reaching the final configuration. The cable pretension adjustment also



**Fig. 2.** Bridge details (unit in metre)

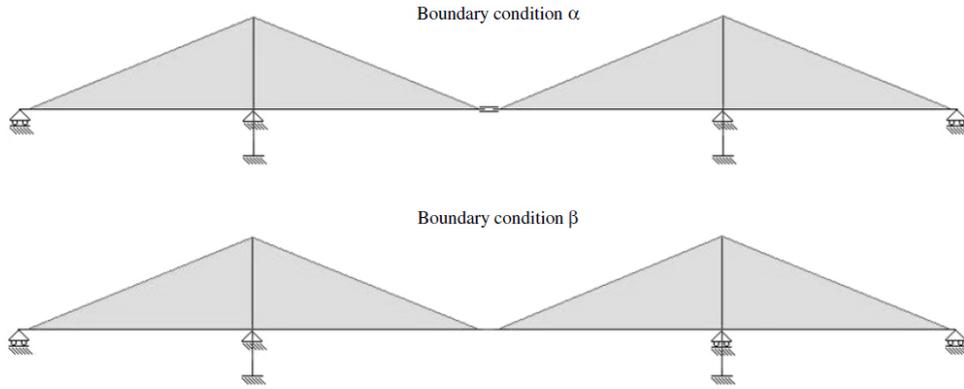


Fig. 3. Boundary conditions

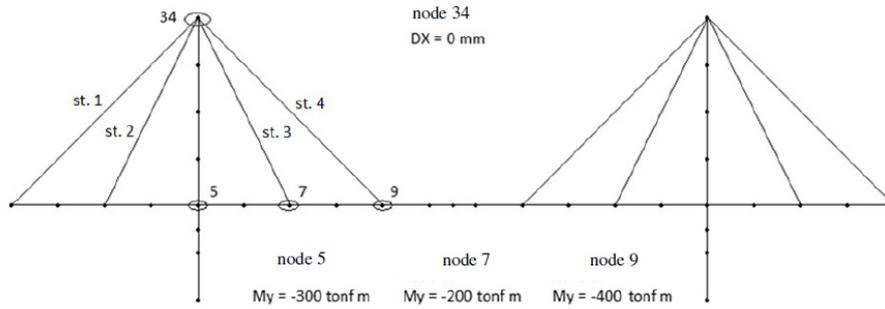


Fig. 4. Imposed conditions on the bridge structure

during every construction phase is hard and expensive to be applied hence a procedure that could determine the stress level of the cable for the installation phase and the final phase is useful. Two alternatives are available: the step back and the step forward analysis: in the first case, the procedure goes back vs. the building construction operations, whereas in the second case the analysis follows the operations. Finally, the pretension of every cable is determined both at the construction phase and at the end: moreover, a final configuration of the structure can be evaluated, and every construction step is verified for safety. The structural response could be determined by non linear analysis for materials, or non linear analysis for geometry. Both analyses have been performed and are reported in the following.

#### 2.4 Step back analysis

This analysis could be understood by observing Fig. 7, in which are illustrated the various steps:

- $step(n-1)_g$ : the DC segments is built, and the other members are loaded with this new load;
- $step(n-1)_c$ : the  $n-1$  cable is built and tensioned at the  $Y_{n-1}$  initial force;
- $stepn_g$ : the BC segment is built as the first step;
- $stepn_c$ : the  $n$ th cable is installed and tensioned with the initial force  $Y_n$ ;
- $step(n+1)_g$ : the segment until the mid-span is built;
- $step(n+1)_c$ : key segment are built, eventually applying the

$Y_{n+1}$  bending moment in relation to the force distribution that it is intended to be realized;

$step(n+2)$ : structure is completed by applying the final permanent loads (pavement and accessories).

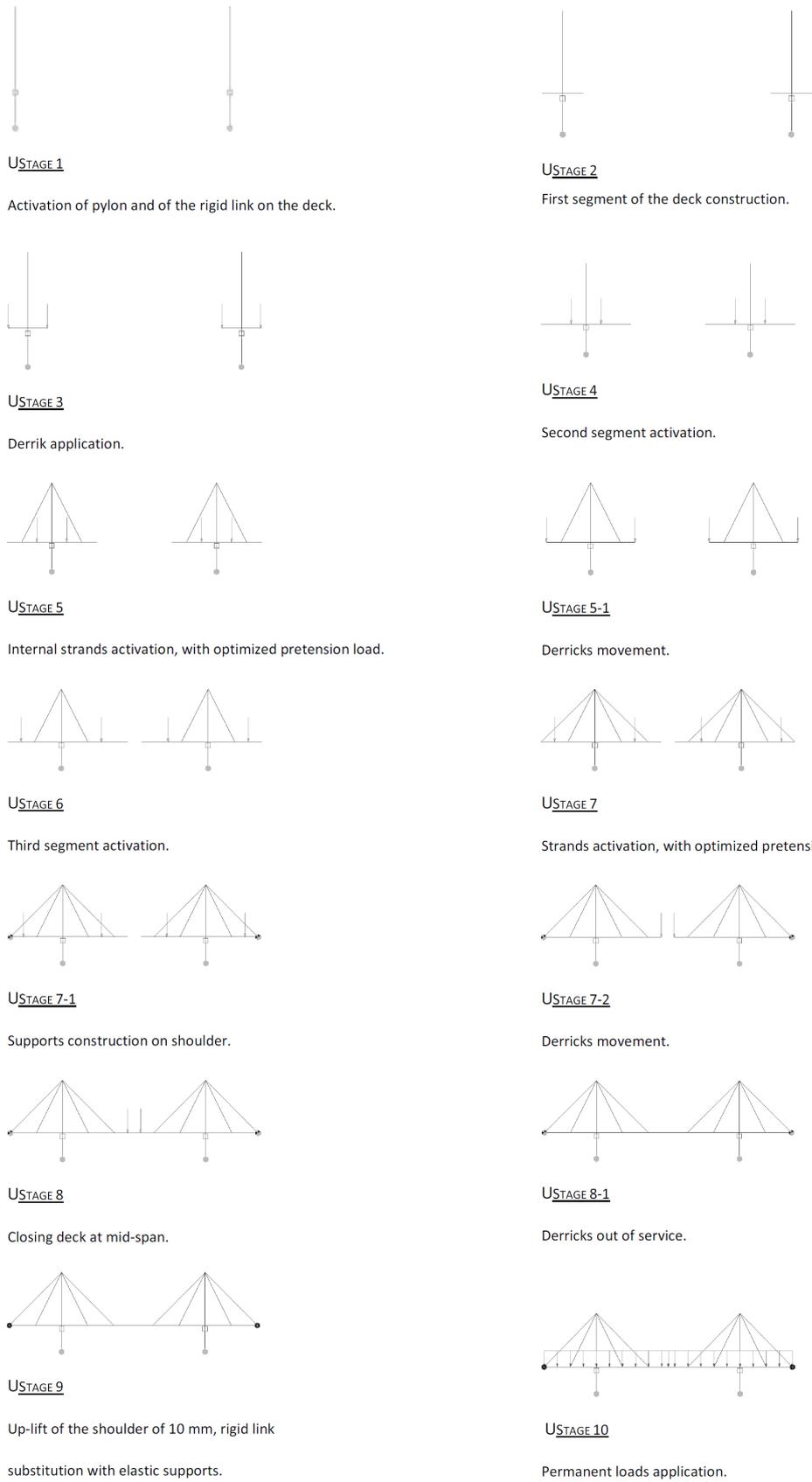
At the end, a specific configuration has to be reached in relation to a particular distribution of forces  $T$  and moments  $M$  ( $T_{1,0}, T_{2,0}, \dots, T_{n,0}, M_{n+1,0}$ ). This condition is known, but the cable initial pretension forces ( $Y_1, Y_2, \dots, Y_n$ ) and the induced moment at mid-span ( $Y_{n+1}$ ) are unknown. This problem could be solved by considering an inverse analysis sequence, calculating the structure from a known configuration and progressively “demolishing” the structure in the same order of the previous passages. The following steps are defined:

$step1$ : the permanent final load is removed ( $g_s$ ), applying a  $-g_s$  load:

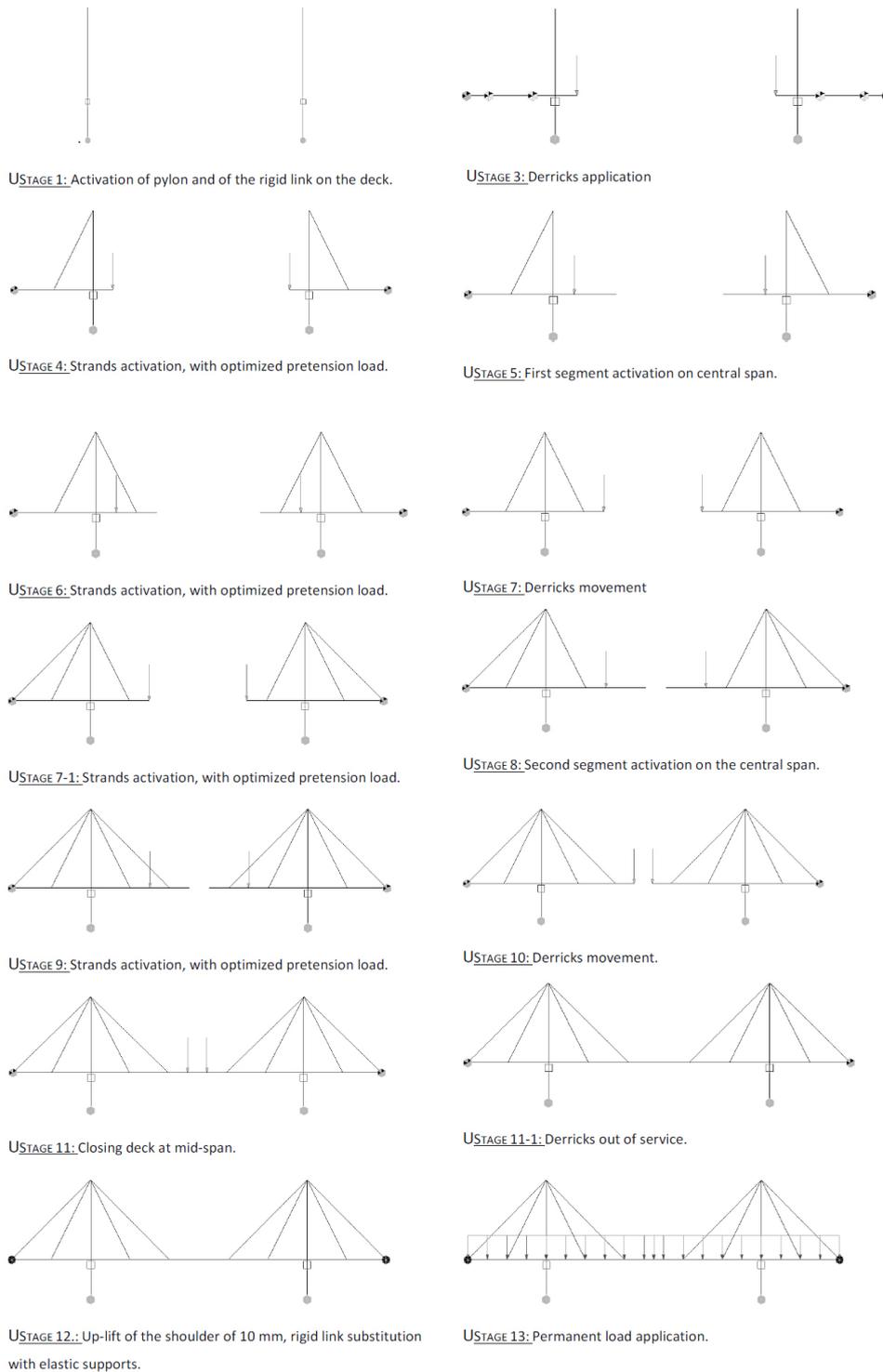
$$\begin{aligned} & (T_{1,1}; T_{1,2}; \dots; T_{n,1}; M_{n+1,1}) \\ & = (T_{1,0} + \Delta T_{1,1}; T_{2,0} + \Delta T_{2,1}; \dots; \\ & \quad T_{n,0} + \Delta T_{n,1}; M_{n+1,0} + \Delta M_{n+1,1}) \end{aligned}$$

which are the parameters of the  $(n+1)_c$  phase; the moment  $Y_{n+1}$  to be applied at mid-span before the closure of the mid-span segment should be  $M_{n+1,0} + \Delta M_{n+1,1}$ ;

$step2_c$ : the central joint at mid-span is removed, and the bridge is build up by two independent components; subjected to the moment  $-Y_{n+1}$  acting at the free ends and equal to  $-(M_{n+1,0} + \Delta M_{n+1,1})$ ;



**Fig. 5.** Construction phase, double cantilever method



**Fig. 6.** Construction phase, lateral cantilever method

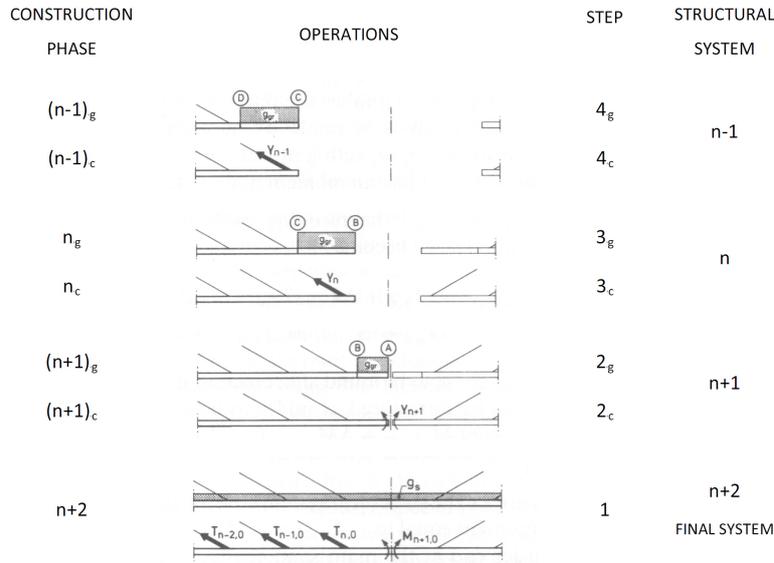


Fig. 7. Moving load due to the movement of the derrick

*step2<sub>g</sub>*: the same structure defined at the previous point is defined, subjected to a equal and contrary load of the dead load of the segment A-B ( $g_r$ ); in this way the cable force is:

$$(T_{1,2}; T_{2,2}; \dots; T_{n,2};) \\ = (T_{1,0} + \Delta T_{1,1} + \Delta T_{1,2}; \dots; T_{n,0} + \Delta T_{n,1} + \Delta T_{n,2}) \quad (4)$$

which are the parameters of the ending phase  $n_c$ . The cable force ( $Y_n$ ), to be introduced in the element at the moment of installation is equal to  $T_{n,2} = T_{n,0} + \Delta T_{n,1} + \Delta T_{n,2}$ ;

*step3<sub>c</sub>*: the partial structure is considered, without the segment A-B and the nth cable, subjected to a force  $-Y_n$ , equal to  $-(T_{n,0} + \Delta T_{n,1} + \Delta T_{n,2})$ , acting in the anchoring point of the removed strand;

*step3<sub>g</sub>*: the same structure of the previous point is considered, subjected to a load equal and contrary to the dead load of the B-C ( $g_r$ ) segment, hence the cable force is:

$$(T_{1,3}; T_{2,3}; \dots; T_{n-1,3};) \\ = \left( T_{1,0} + \sum_i^3 \Delta T_{1,i}; \dots; T_{n-1,0} + \sum_i^3 \Delta T_{n-1,i} \right) \quad (5)$$

which corresponds to the forces at the end of the  $(n-1)_c$  phase; the cable force ( $Y_{n-1}$ ) to be introduced in the element at the moment of installation is equal to .

$$T_{n-1,3} = T_{n-1,0} + \Delta T_{n-1,1} + \Delta T_{n-1,2} + \Delta T_{n-1,3} \quad (6)$$

This procedure has to be repeated until the complete “demolition” of the structure, and the initial force on each cable can be computed. Another key aspect to be considered is related to the moving load applications during erection, for e.g. the derrick movement: at the  $(n-1)_g$  step, the derrick weight ( $P_{cr}$ ) is applied at the free ends of the deck where the successive segment is lifted up; at the  $n_g$  step, the derrick will be moved across the

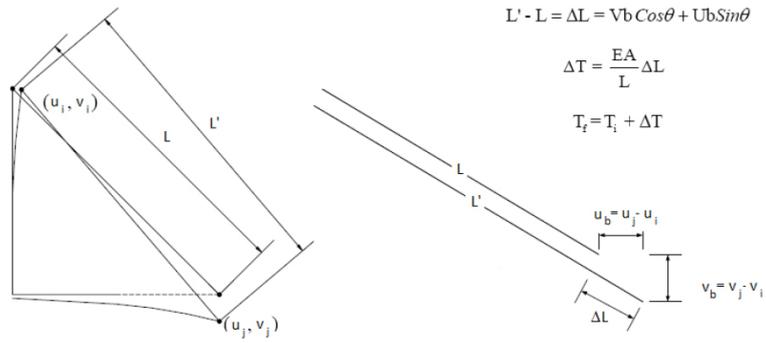
new segment, the weight of the derrick is now applied to this new free ends, and in correspondence with the previous point is applied a  $-P_{cr}$  load, which is able to led to null the derrick weight of the step  $(n-1)_g$ .

### 2.5 Step forward analysis

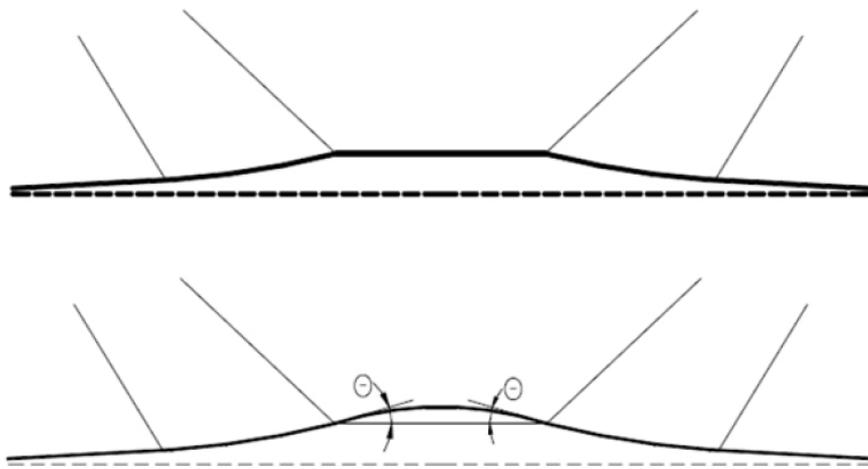
The step forward analysis follows the construction phases. The most difficult operation is to define the exact tensioning level of each cable with the aim of obtaining a determined configuration at the end of construction. This could be obtained with a step back analysis, or calculating for each construction phase a pretension level to be applied when the cables are active and an eventual moment to be applied at the end of the erection. This second approach is implemented in Midas (2000) with the special function “Lack of Fit Force”. During the erection cable anchoring are in a particular deformed configuration, hence a  $\Delta T$  force is required to locate the element and fitting the structure in the desired position; this has to be added to the pretension calculated for the final configuration of the bridge ( $T_i$ ). This value is related also to the initial ( $L'$ ) and final ( $L$ ) length of the cable, as reported in Fig. 8-9. The Lack of Fit Force function is used at the moment of the closing key mid-span segment: it is necessary to define a force system to be applied at the free ends of the concurring segments to establish the continuity of the deck curvature at the end of the construction.

### 3 Results

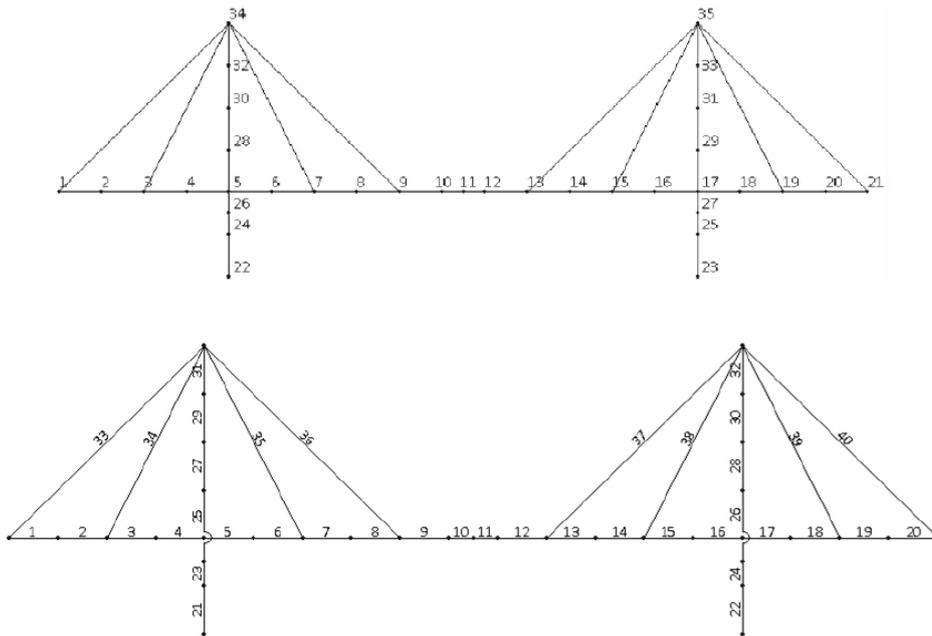
The structural response has been studied, for both previously described complete and partial cantilever construction processes. The structural analyses, have been conducted in both the linear (LSA) and non-linear field, with non linearity in cables' behavior (NLSA<sub>lg</sub>) and geometric non linearities (NLSA<sub>nlg</sub>). The construction process has been studied with



**Fig. 8.** Lack of Fit Force calculation in a cable



**Fig. 9.** Lack of Fit Force calculation in the key mid-span segment



**Fig. 10.** FEM reference for the model, nodes and elements numerations

regard to both step forward and step-backward analysis. In Figure 10 numbering of nodes and elements is shown. As previously described, the pretension levels in the cables are optimized with a quadratic objective function and the already specified boundary condition. The initial shape of the bridge can be determined in a single phase (final stage), or as a final phase of a given backward or forward stage construction process. In the following, the results of the various analyses are presented with reference to the axle loads in cables, vertical displacements and bending stresses in the girder for comparison purposes.

All the seven analyses give the same results (Figures 11-13): this could help to infer that the different procedures are accurately used, as the same results are obtained, however it should be considered that this is strictly dependent on the structural arrangement and the applied assumptions in the non-linearities. In Tab. 4 the numerical values are reported. In Tab. 5 are presented the results obtained with a NLSA\_lg and a NLSA\_nlg analysis. In the partial cantilevered construction phase method, the central span is built starting from the pylons and the partial spans supported by temporary restraints.

As illustrated in Fig. 14, deck segments are added towards the central span; simultaneously, cables are installed to limit deck displacements, proceeding alternatively with the partial and the central span, from stage 1 to stage 10; the closing segment is then built in phase 11 at mid-span; once completed the deck, provisional equipment is removed and pavement is realized, as described in phases from 11 to 13. Axle load, displacement at the top of the pylon both in horizontal and vertical direction, vertical girder displacement, axial and bending stress in various section of the pylon and of the girder are shown in Figs. 15-17 for the step back analysis.

Axle load in the cables in the internal strands (n. 34 and 35) is smaller than in the external cables (n. 33 and 36). Due to unbalancing of the structure during the construction of the partial span, stress oscillates before reaching the final value. The horizontal displacement of the top of the pylon (node 34) and of a cross-section at the superior part of pylon (node 32) is directed towards the partial span, also in this case with an oscillatory behavior, until becoming null in the final configuration. The vertical displacement of the node corresponding to the abutment (node 1), once defined the partial span, resulted to be approximately 10 mm; the vertical displacement of the node at the centre of the partial span oscillates around positive values, once removed the temporary supports; also the vertical displacement of the nodes of the central span has an oscillatory behavior, due to unbalancing of the structure during cantilevering. The pylon resulted to be subjected to an axial-bending state of stress during all the construction phase: in detail the axial forces monotonically increase both in the lower (node 22) and upper part (node 28) when construction phases advance, whereas the bending moment has an oscillatory behavior. Also the deck is subjected to axial forces and bending moment: the axial force increase when the construction advances with peak values at strand ac-

tivation, for e.g. in partial span at stage 4 and 7-1, and central span at stage 6 and 9. Furthermore, the central span shows a reduction of the solicitation at stage 11 when the closing segment is applied at mid-span. The bending moment has an oscillatory behavior with limited variation in the partial span (node 4), and more significant in the central span (node 6), as shown in Fig. 17. Concerning the forward construction phase, axial force in the cables, horizontal displacement at the top of the pylon, vertical displacement of the girder, axial force and bending moment in the pylon and in the girder are shown in Figures 18. Results during the construction phases have confirmed those of the step back analysis, except for the vertical displacement of the nodes of the partial span. In the step back analysis, the vertical displacement at the abutment (node 1) is approximately 10 mm from stage 12 to stage 2; however in the step forward process the vertical displacement of node 1 is approximately zero until stage 12, when the 10 mm displacement is impressed, since this analysis follows the real construction process.

In Fig. 19 and 20 the various configurations assumed during the construction stages are presented by comparing displacements obtained with step forward and step back analysis. It can be noticed a good correspondence between the analyses except between stage 11-1 and stage 2, at which the step back structural behavior is influenced by the uplift of the abutment. By confirming these considerations, results concerning the axial force in cables, and of the vertical displacement of the partial and central span, respectively at the mid-span node and at the one third of the span are shown in Figure 21. In detail, it could be noticed a correspondence of the structural behavior of the central span, while a little deviation could be observed in the partial span. This paragon, has evidenced in particular the difficulty of the step back analysis to evidence the phenomena which appears to be active in the following phase, as the calculation procedure of the transient structure is carried on by starting from the final phase. This limit could be exceeded by analyzing the stages in the same direction with respect to construction, so the effect is only on later stages. The technique of the full cantilevered method allows to build the complete deck from the pylons: segments are added first on one side then from the other, adding the correspondent cables to limit the vertical displacement. By comparing the partial and the full cantilevered method limits and positive issues of these technique can be observed. The step forward analysis follows exactly the construction process, from pylons erection, to girder execution and suspension erection until the final stage of the complete structure under the permanent loads action.

**Tab. 3.** Pretension forces

	Pretension [tonf]
Cable 1	336.9
Cable 2	254.0
Cable 3	193.3
Cable 4	341.0

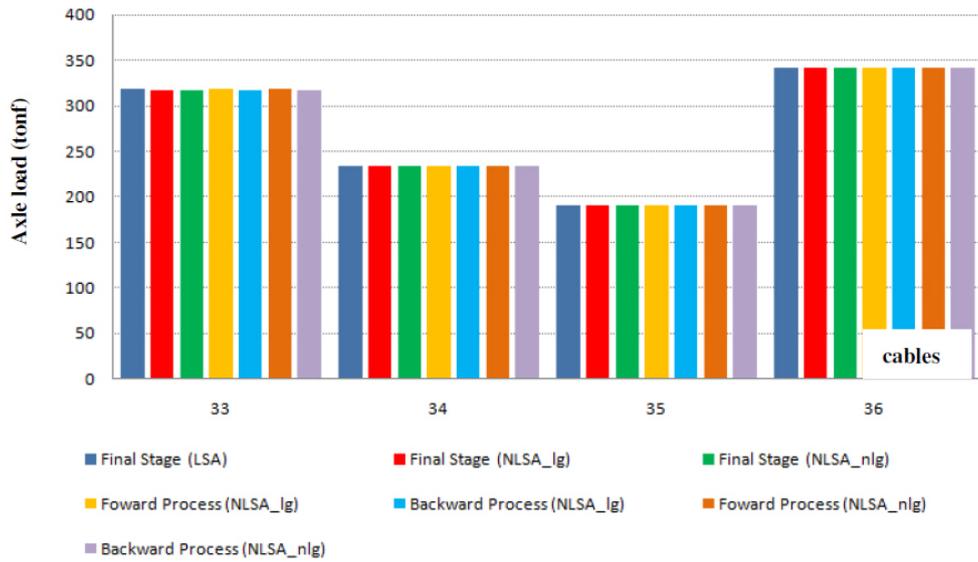


Fig. 11. Axle load in cables, partial cantilever construction mode (1 tonf=10 kN)

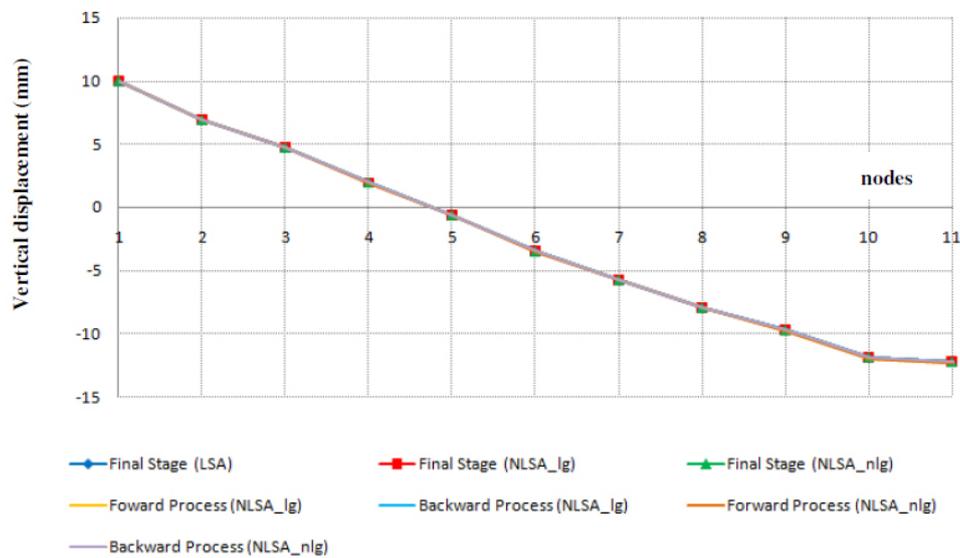


Fig. 12. Vertical displacement in the control points of the girder, partial cantilever construction mode

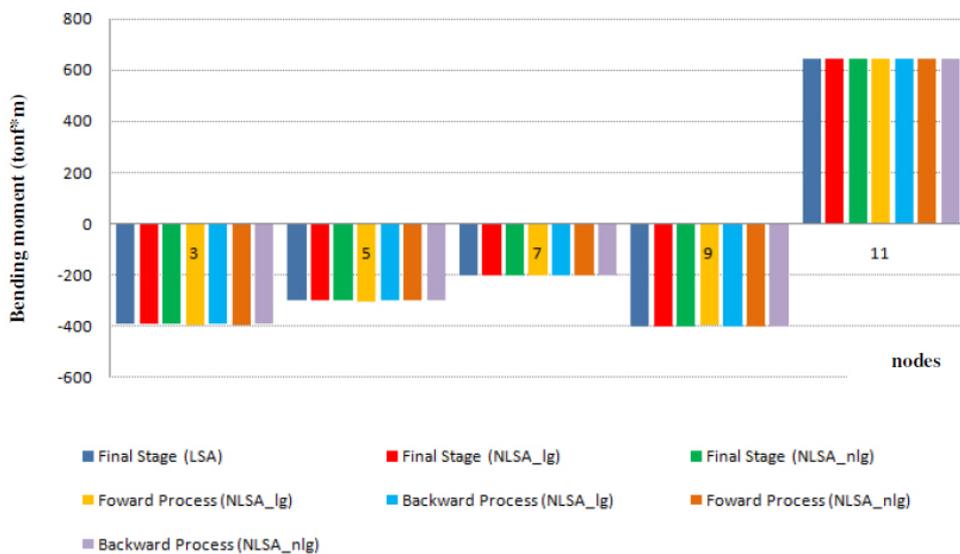
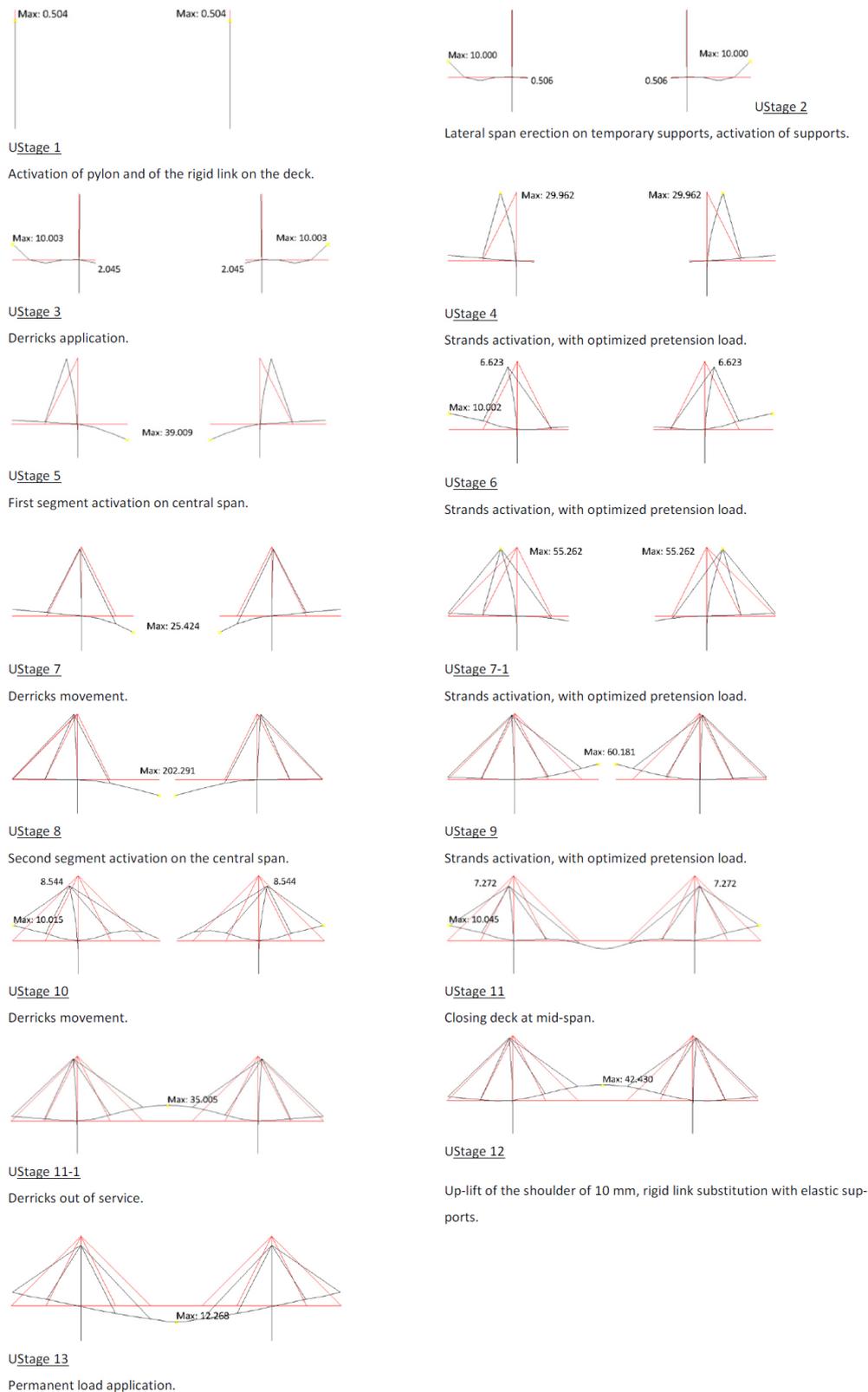


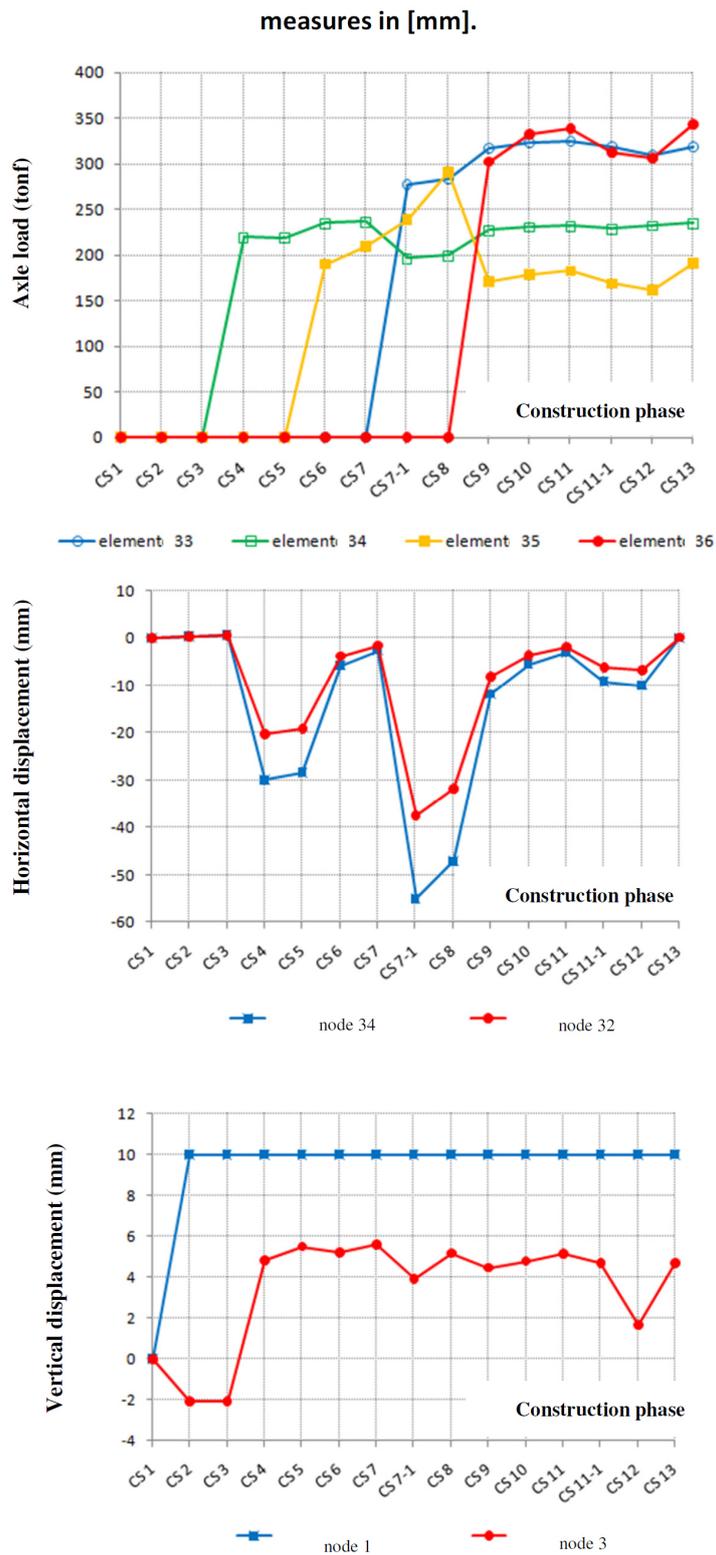
Fig. 13. Bending moment in the control points of the girder, partial cantilever construction mode (1 tonf=10 kN)



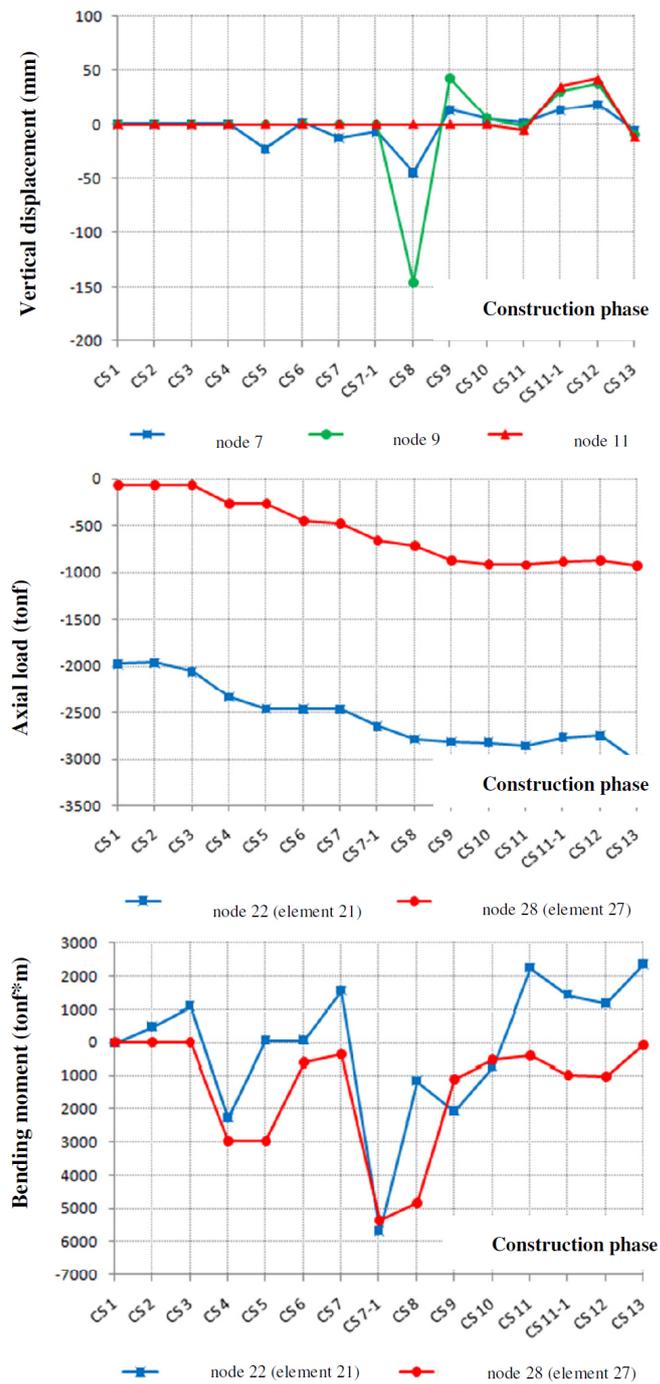
**Fig. 14.** Structure configuration during the partial cantilevered construction phases (BPA)

**Tab. 4.** Axle load in cables (tonf) (1 tonf=10 kN)

	Final Stage			Backward process		Forward process	
	LSA	NLSA_lg	NLSA_nlg	NLSA_lg	NLSA_nlg	NLSA_lg	NLSA_nlg
cable 33	318.924	318.914	318.863	318.914	318.863	318.994	318.935
cable 34	234.797	234.793	234.751	234.793	234.751	234.840	234.800
cable 35	190.933	190.938	190.934	190.938	190.934	191.009	190.908
cable 36	343.025	343.029	343.039	343.029	343.038	343.098	343.019



**Fig. 15.** Axle load variation in the cables, horizontal displacement variation of the pylon, vertical displacement variation in the lateral span (NLSA\_lg) (1 tonf=10 kN)



**Fig. 16.** Vertical displacement variation in the central span, axle load variation in the pylon, bending moment variation in the pylon (NLSA\_nlg) (1 tonf=10 kN)

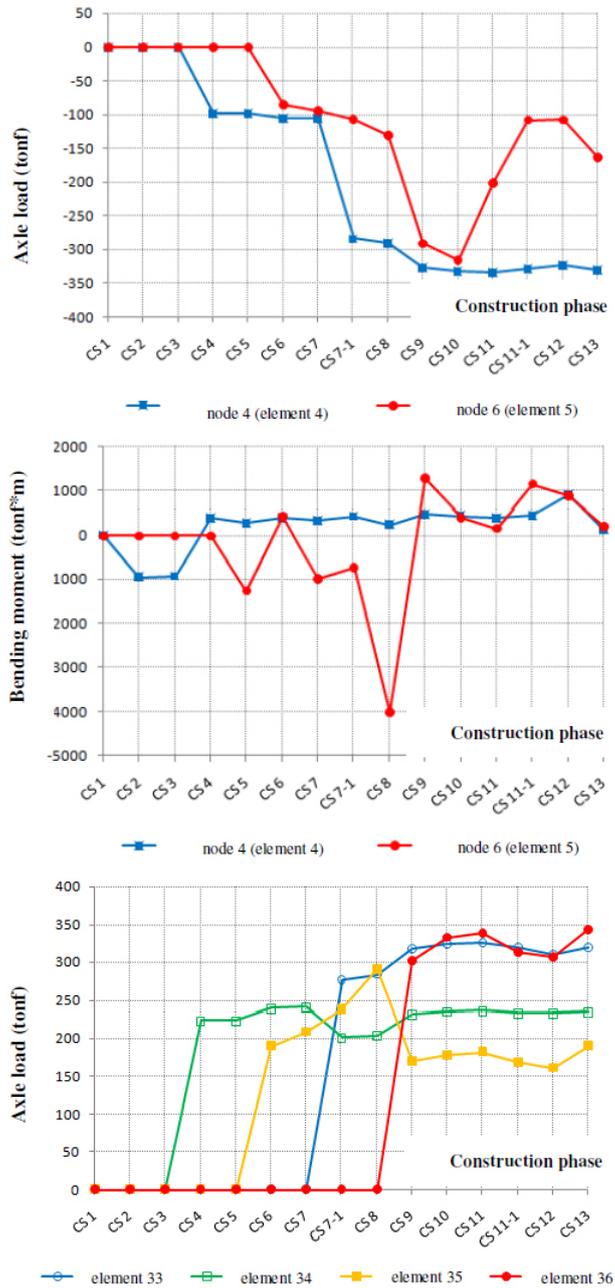


Fig. 17. Axle load variation in the girder, bending moment variation in the girder, axle load variation in cables (NLSA\_nlg) (1 tonf=10 kN).

**Fig. 18.** Horizontal displacement variation of the pylon, vertical displacement variation in the lateral span, vertical displacement variation in the central span (NLSA\_nlg) (1 tonf=10 kN)

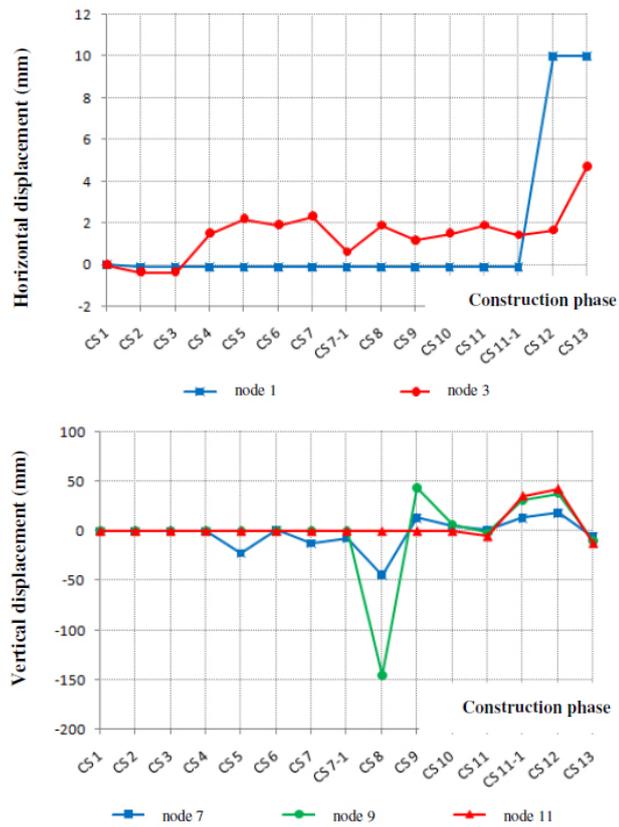
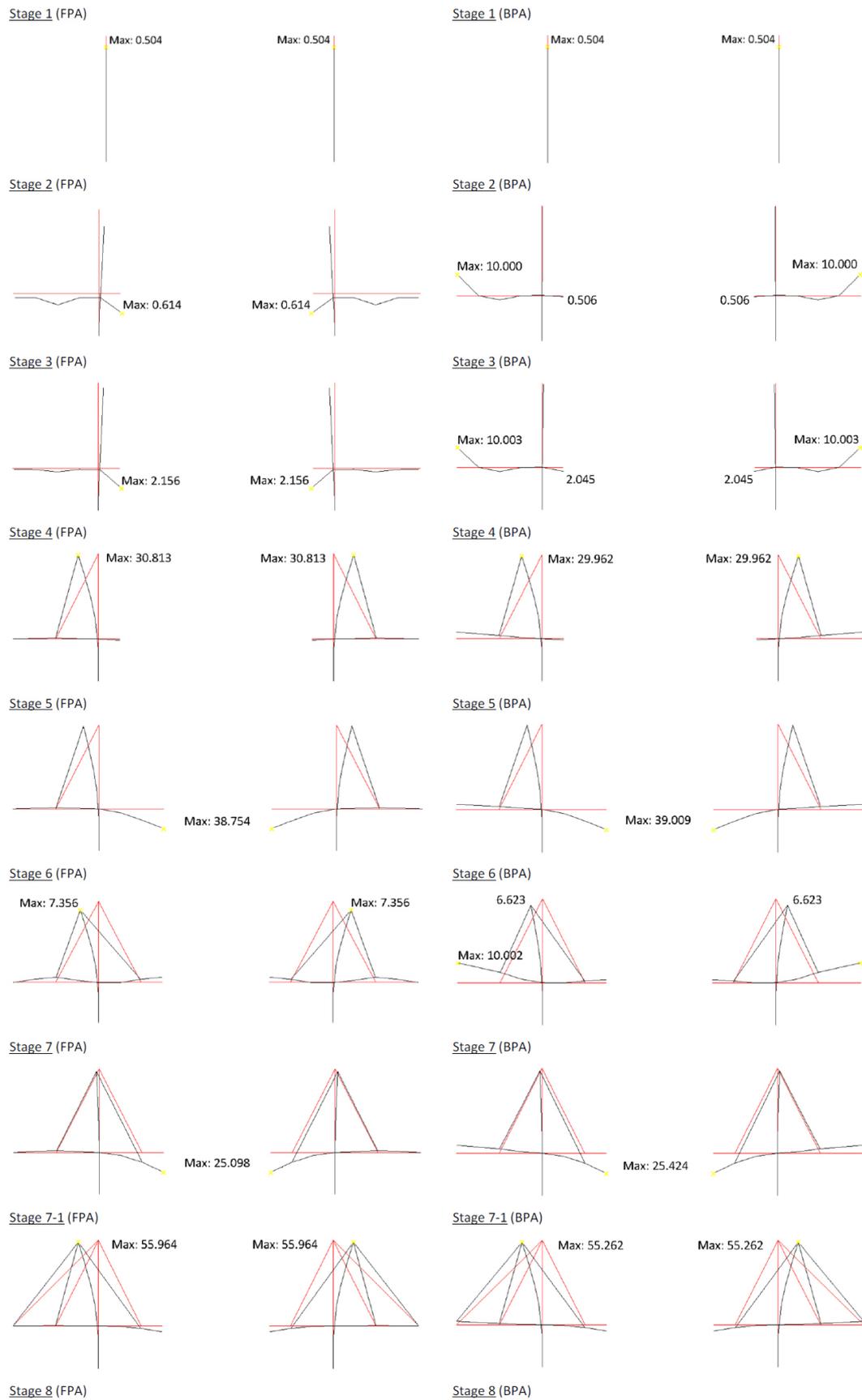


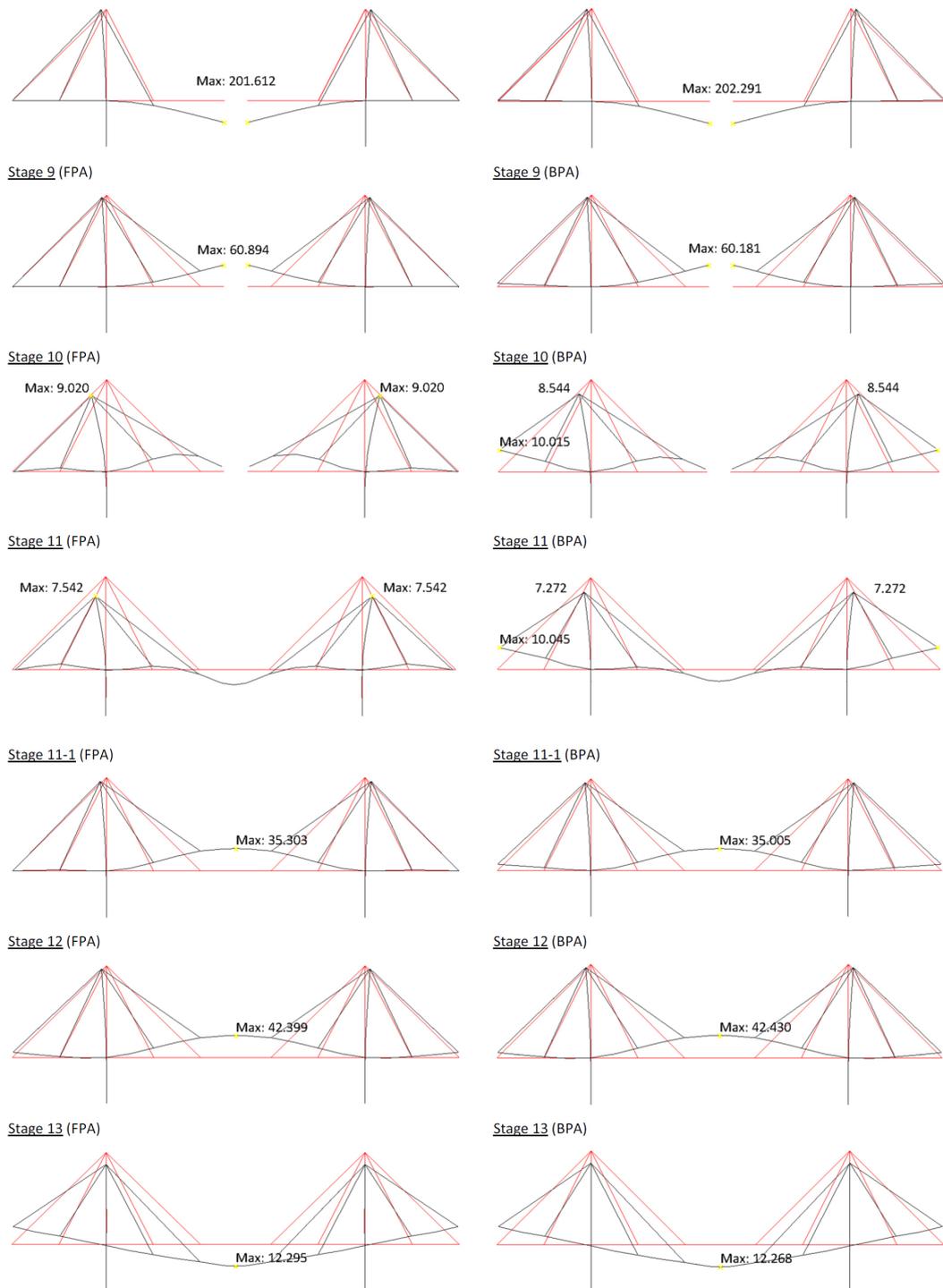
Figure 22 highlights the configurations during the construction process, with a forward analysis and the corresponding displacements. In the following are presented the results obtained in relation to the axle load on cables, the horizontal displacement at the top of the pylon, the vertical displacement of the girders, and the axial and bending forces in some sections of the pylon and the girder (Figs. 23-25).

Axial force in the cables in the internal strands (n. 34 and 35) is smaller than in the external cables (n.33 and 36). Due to unbalancing of the structure during the construction of the partial span, stress oscillates before reaching the final value. The horizontal displacement at the top of the pylon and at a section of the upper part of the pylon (node 32) has a oscillating behavior until zero. The full cantilevered method is more balanced if compared to the partial one since the displacement of the pylon is smaller. The vertical displacement of the node at the abutment (node 1) and the node of the central and partial span (node 3) has a behavior influenced by the cables activation. In detail, after the external cables' activation (stage 7) this value is constant until stage 9 at which the abutment displacement is fixed at 10 mm. The vertical displacement of the nodes of the central span has a oscillatory behavior due to the unbalancing of the structure during cantilevering in the central span. The pylon is subjected to bending moment and axial force during the whole construction. In detail, the axial force grows up monotonically both in the lower (node 22) and in the upper (node 28) part during construction, whereas the bending moment has an oscillating behavior; however the excursion of this values appears to

be smaller than the cantilevered partial construction. The deck is also subjected to axial force and bending moment: the axial force grows up with construction, with higher values during cables activation in stages 5 and 7; a reduction in the stress at stage 9 can be observed in the central span; the bending moment has an oscillating behavior, with a similar excursion in the partial span at node 4 and in the central span at node 6. After determining the structural behavior of the partial structure with the forward process, the partial or single and the full or double cantilevered method, the results were compared. By comparing the structural response determined taking into account the construction phases and the response of the bridge without the construction phases (at the final stage) a good agreement of the solutions related to cables forces and vertical displacement of the girder nodes is observed in Figs. 26 and 27. The comparison has been extended to the construction phase considering the axial force in the cables, the horizontal displacement at the top of the pylon, the vertical displacement of the girders nodes and the bending moment in some sections of the pylon and the girder (Figs. 28-30). The full or double cantilevered construction method seems to be the most balanced; this preliminary analysis considering the analyzed data, could allow to infer that this method suggest a more convenient dimensions of the pylons, even if some more analysis has to be performed to confirm this conclusion. For the other structural elements, the various construction processes seem to be substantially similar.



**Fig. 19.** Structure configuration during the partial cantilevered construction phases, measures in [mm]: comparison between back (BPA) and forward analysis (FPA) (Stages 1-7)



**Fig. 20.** Structure configuration during the partial cantilevered construction phases, measures in [mm]: comparison between back (BPA) and forward analysis (FPA) (Stages 8-13)

**Tab. 5.** Pretension of the cables at the initial phase, by using the LFF option (1 tonf=10 kN)

	NLSA_lg		NLSA_nlg	
	TR <sub>r</sub> R [tonf]	ΔT [tonf]	TR <sub>f</sub> R [tonf]	ΔT [tonf]
cable 33	336.992	-7.083	329.839	-7.032
cable 34	253.976	0.120	254.096	0.043
cable 35	193.309	51.580	244.889	55.129
cable 36	340.997	180.059	521.056	185.298

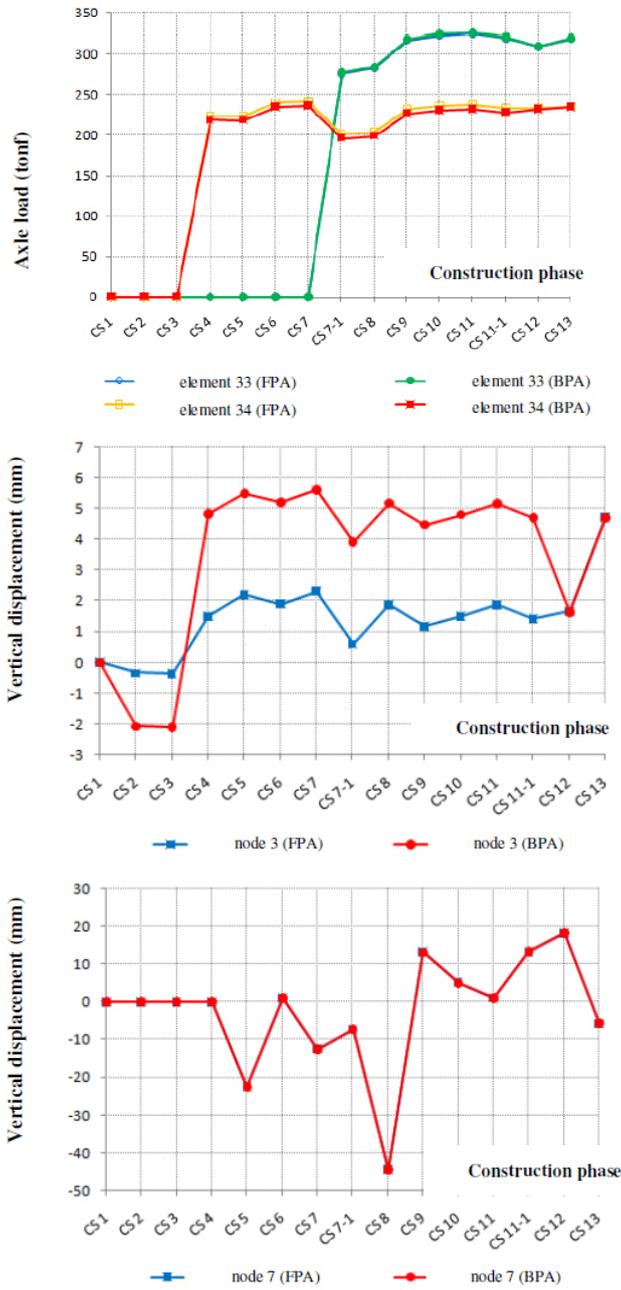
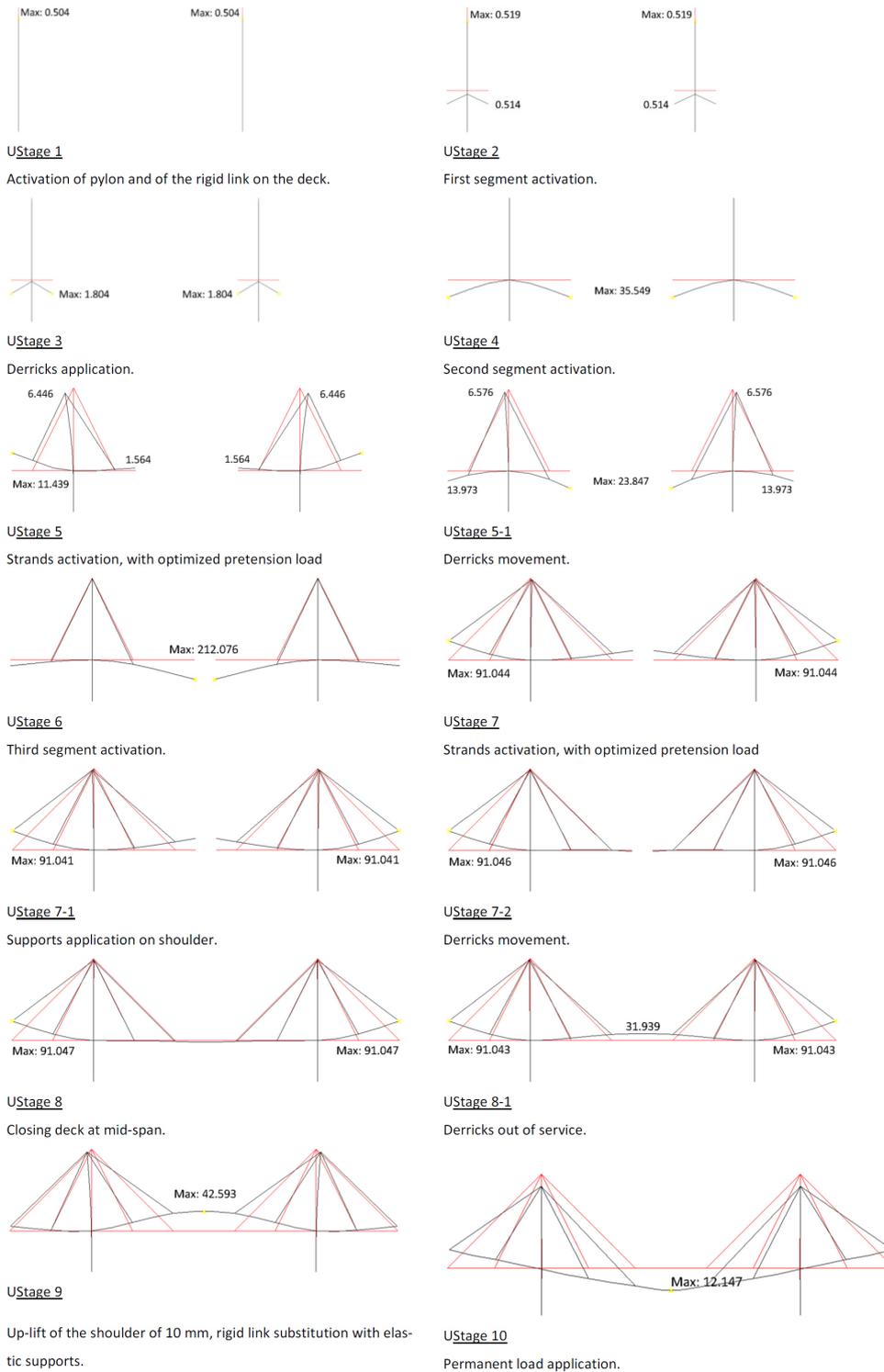
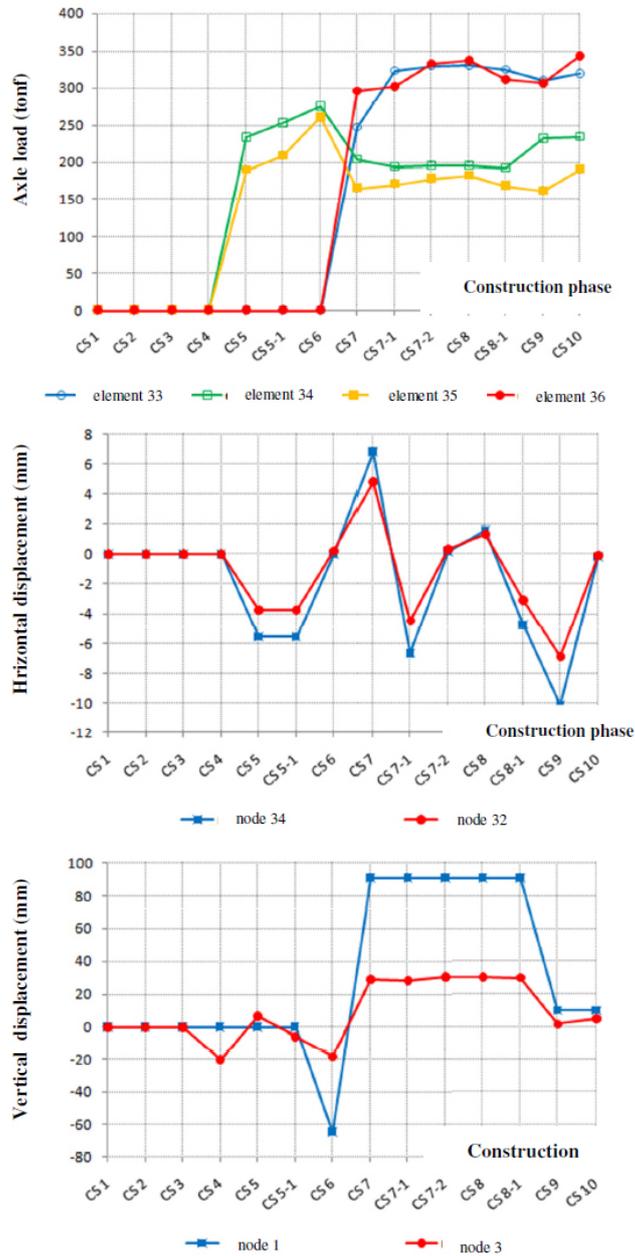


Fig. 21. Axle load variation in cables, node 3 variation of the vertical displacement, node 7 variation of the vertical displacement (NLSA\_nlg) (1 tonf=10 kN)



**Fig. 22.** Structure configuration during the total cantilevered construction phases, measures in [mm]



**Fig. 23.** Axle load variation in cables, horizontal displacement variation of the pylon, vertical displacement variation of the lateral span (NLSA\_nlg) (1 tonf=10 kN)

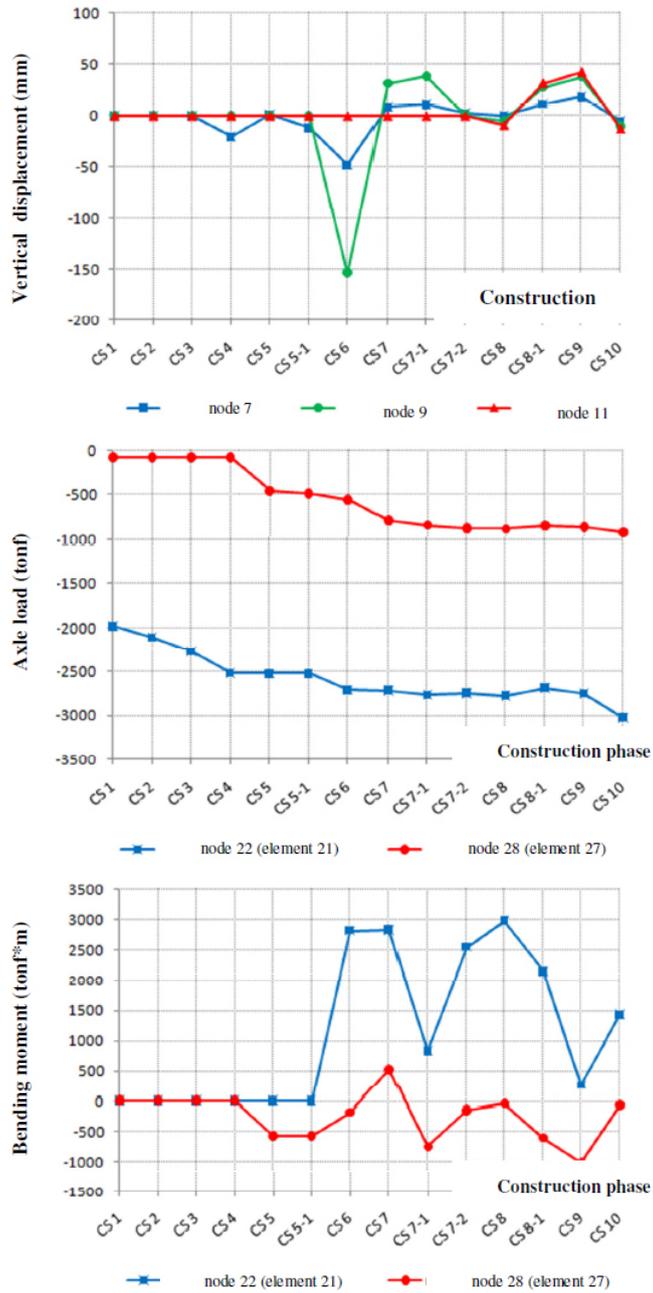


Fig. 24. Vertical displacement variation of the lateral span, axle load variation in cables, bending moment variation in the pylon (NLSA\_nlg) (1 tonf=10 kN)

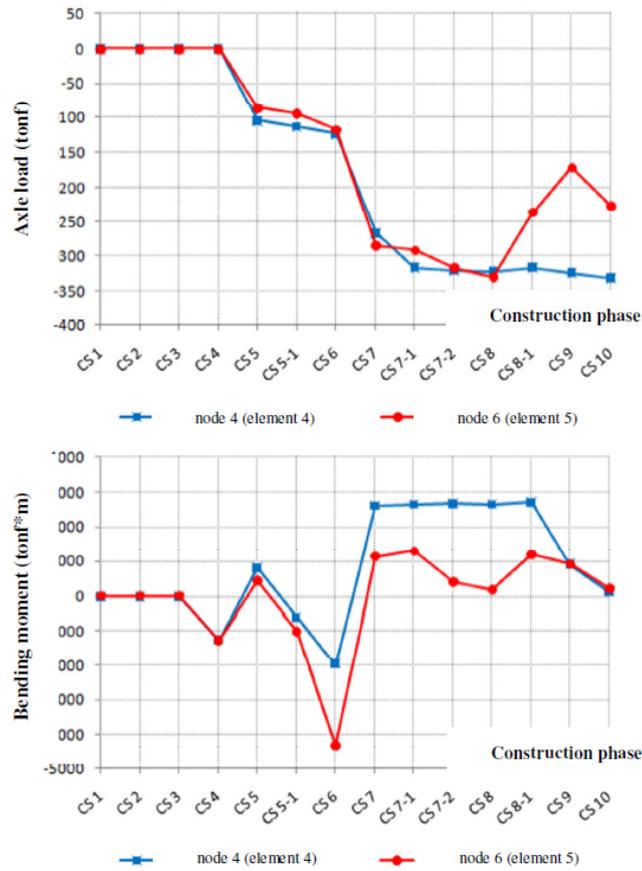


Fig. 25. Axle load variation in the girder, bending moment variation in the girder (NLSA\_nlg) (1 tonf=10 kN)

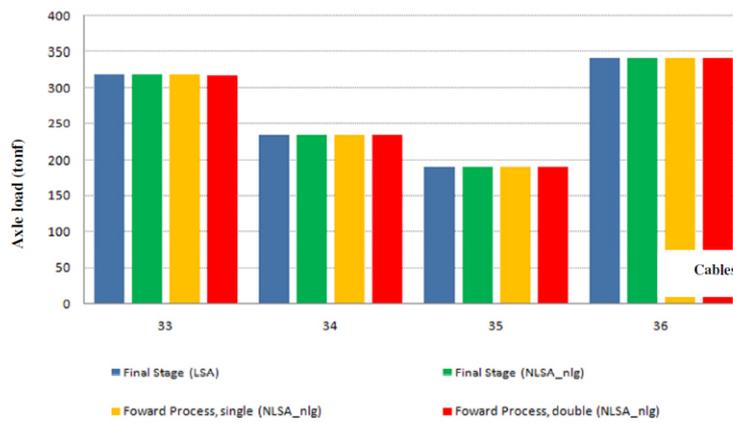


Fig. 26. Axle load in cables (1 tonf=10 kN)

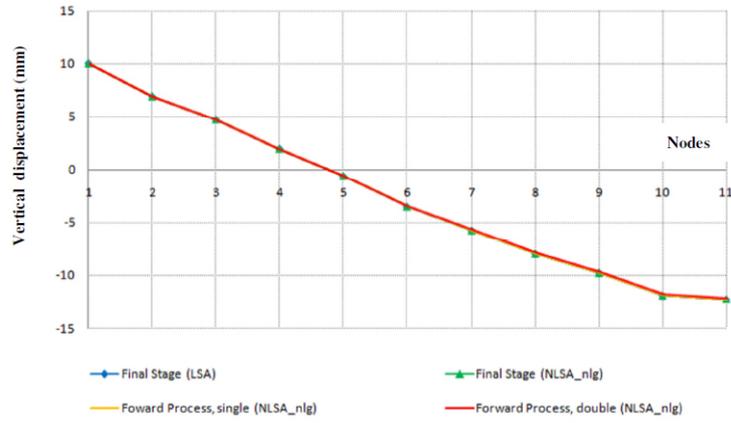


Fig. 27. Vertical displacement in the control nodes of the girder

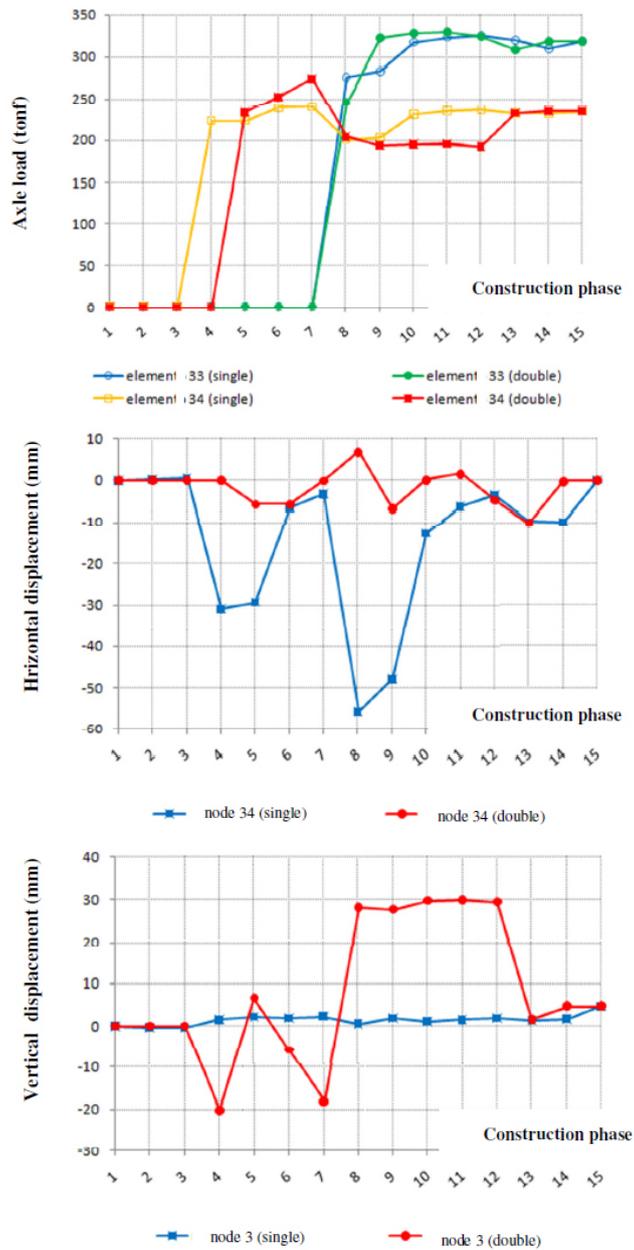
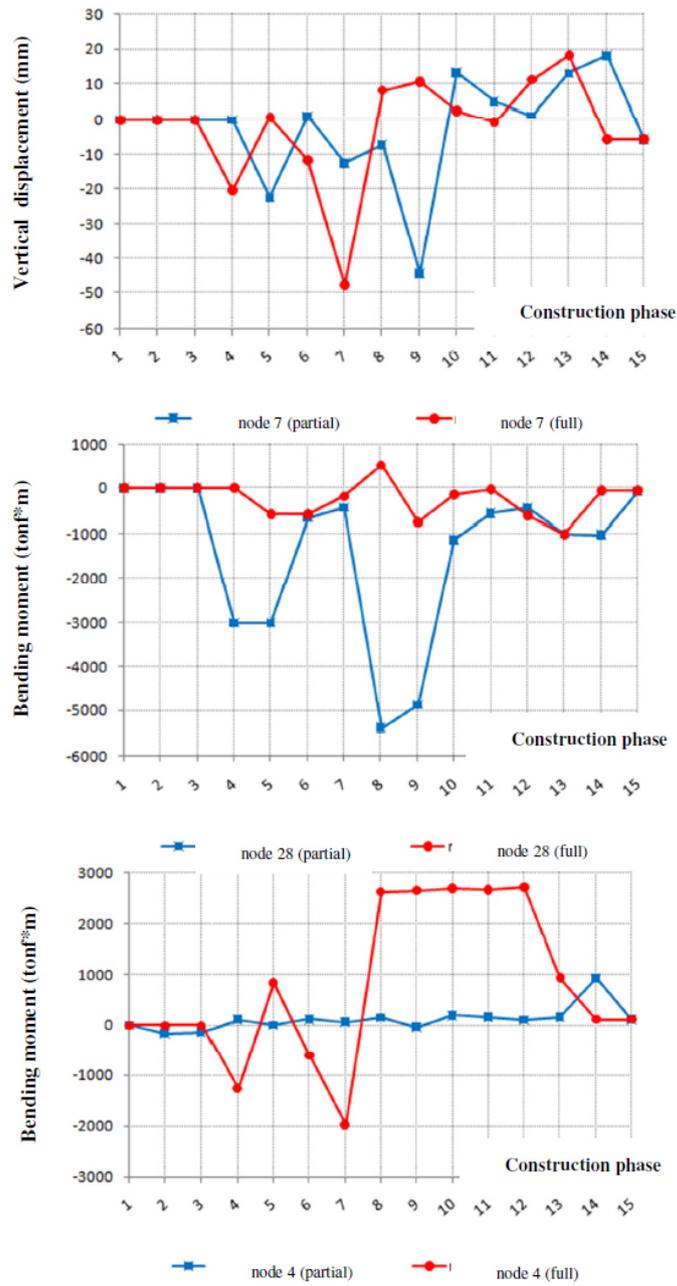
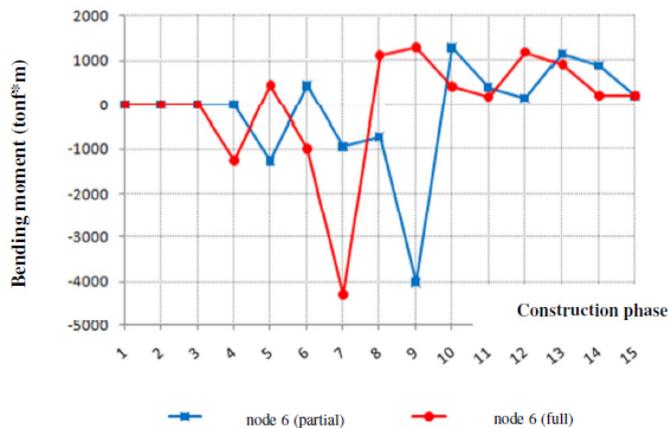


Fig. 28. Axle load variation in cables, Horizontal displacement variation in the pylon, Vertical displacement variation in the lateral span (NLSA\_nlg) (1 tonf=10 kN)



**Fig. 29.** Vertical displacement variation in the central span, bending moment variation in the superior part of the pylon, bending moment variation in the lateral span (NLSA\_nlg) (1 tonf=10 kN)



**Fig. 30.** Bending moment variation in the central span (NLSA\_nlg) (1 tonf=10 kN)

## 4 Conclusions

A finite element computational procedure is set up for a construction process comparison of cable-stayed bridges during erection procedures. Different construction cantilever methods are compared, the partial and the full cantilever method, by adopting different process analysis, and at the same time comparing either the linear computation procedure (linear theory) or the nonlinear computation procedure (nonlinear theory). Different computational process are established: the step forward process analysis and the step backward process analysis. The forward process analysis of cable-stayed bridges during construction is performed by following the sequence of erection stages in bridge construction, while the backward process analysis follows the reverse direction of the sequence of the bridge erection procedure. At each erection stage, the finite element analysis model is rebuilt, then the system equation is set up and solved. Concerning the construction method, it is confirmed that the full cantilevered method is more balanced if compared to the partial one, thus suggesting that this method is more convenient in the pylons dimensioning. About process type, and basing on the numerical analysis performed, some general conclusions could be inferred: it is confirmed that both the forward and backward methods can be used for finding the configurations and pre-forces in members of the bridge structure at different erection stages during the girder construction using the cantilever method; it is confirmed that both the forward and backward methods can be used successfully for the partial and the full cantilever method; as a novel observation, the step back analysis seemed to be more limited in representing the phenomena of the following subsequent stages, since the transient calculation of the structure was performed starting from the final configuration: this limit can be avoided by analyzing the stages of the process in the same direction with respect to construction, as in the forward analysis every phenomenon highlights its effects only in the successive stages without affecting the previous ones; while the solution of the step back method offers the accurate configuration and member pre-forces of the bridge structure at different erection stages, the solution of the step forward method is not unique, since iteration is carried out at erection stages, thus implying that the numerical results depends on the estimated cable initial forces used for starting the computation of shape iterations. Considering the analysis type as task dependent results related to this particular case study analysis, an agreement between the results of the linear and non-linear analyses has been observed: in both the partial and full cantilever method, the nonlinear theory offers theoretically more accurate results than that determined by the linear theory, even if the computation becomes more complicated and time-consuming when the nonlinear theory is utilized.

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